

## Curriculum Vitae

### SHRAWAN KUMAR

Department of Mathematics  
University of North Carolina at Chapel Hill  
Chapel Hill, NC 27599-3250, USA

Telephone: 919-962-9615, Telefax: 919-962-2568  
e-mail: [shrawan@email.unc.edu](mailto:shrawan@email.unc.edu)

#### Academic Qualifications:

**B.Sc.** Gorakhpur University 1973  
**M.Sc.** Bombay University 1975  
**Ph.D. (Maths.)** Bombay University 1986 (Thesis Adviser: S. Ramanan)

#### Positions :

1975-76, **Visiting Member**, Tata Institute of Fundamental Research (Bombay)  
1976-82, **Research Assistant**, Tata Institute of Fundamental Research  
1982-85, **Research Associate**, Tata Institute of Fundamental Research  
Aug. 1983-July 84, **Postdoctoral Research Fellow**, Mathematical Sciences Research Institute (Berkeley)  
July 1984-Aug. 85, **C.L.E. Moore Instructor**, Massachusetts Institute of Technology (Cambridge)  
1985-89, **Fellow**, Tata Institute of Fundamental Research  
Sept. 1988-May 89, **Member**, The Institute for Advanced Study (Princeton)  
1989-92, **Reader**, Tata Institute of Fundamental Research  
2006- , **Adjunct Professor**, Tata Institute of Fundamental Research  
1991-2010 , **Professor**, University of North Carolina at Chapel Hill  
2010- , **John R. and Louise S. Parker Distinguished Professor**, University of North Carolina at Chapel Hill

#### Visiting Positions :

- (1) Visiting Scholar at Massachusetts Institute of Technology (Cambridge) for two months April-May 1987
- (2) Visiting Professor at The University of British Columbia (Vancouver) for three weeks during June 1989
- (3) Visiting Professor at University of P. and M. Curie (Paris) for two weeks during May 1990
- (4) Visiting Professor at Scuola Normale Superiore, Pisa (Italy) for one month in May 1991 (to attend the International Semester on **Infinite dimensional algebra and algebraic geometry**).

- (5) Visiting Professor at Ecole Normale Supérieure (Paris) for two weeks during June 1991; three weeks during May-June 1992; one month during May 1994; and one month during May-June 1999
- (6) Visiting Professor at Max Planck Institut für Mathematik (Bonn) for one month during June-July 1993 and again for one week during July 2005
- (7) Visiting Professor at ICTP (Trieste) for one week during April 1994
- (8) Visiting Professor at Tata Institute of Fundamental Research (Bombay) for eight months during Dec. 1993-Aug. 1994; three weeks during July-Aug. 1995; six months from July 2001-Dec. 2001; ten days during January 2005; two weeks during December-January 2006-07; two weeks during December 2008; one week during December 2009; one week during May 2012; one week during December 2012; two weeks during December 2016 - January 2017; two weeks during January 2018; and one week during January 2019.
- (9) Visiting Professor at Research Institute for Mathematical Sciences, Kyoto University (Kyoto) for one month during June-July 1996
- (10) Visiting Professor at the Erwin Schrodinger International Institute for Mathematical Physics (Wien) for two weeks during August 2000
- (11) Senior Visiting Fellow at Isaac Newton Institute for Mathematical Sciences (Cambridge) for six months, January- June 2001
- (12) Visiting Professor at Weizmann Institute (Israel) for two weeks during June 2005
- (13) Visiting Professor at Duke University for one year, August 2006 -July 2007
- (14) Visiting Professor at the Hausdorff Research Institute for Mathematics, Bonn (Germany) for two weeks during January 2011 and for one month during April 2011.
- (15) Visiting Professor at the University of Aarhus, Aarhus (Denmark) for one week during July 2011.
- (16) Visiting Professor at the Harish-Chandra Research Institute, Allahabad (India) for one week during December 2013.
- (17) Visiting Professor at Institut Mittag-Leffler, Djursholm (Sweden) for one month during April - May, 2015 to participate in the 2015 Spring term research programme 'Representation Theory'.
- (18) Visiting Scholar at the University of Sydney, Sydney (Australia) for one semester, July 2015 - October 2015.

#### **Honors and Awards :**

- (1) Principal Investigator NSF grant no. DMS-9203660 for the period 1992-1995
- (2) Principal Investigator NSF grant no. DMS-9622887 for the period 1996-2000
- (3) Principal Investigator NSF grant no. DMS-0070679 for the period 2000-2005
- (4) Granted the W.N. Reynolds competitive leave of absence from the University of North Carolina at Chapel Hill for the period July 1-December 31, 2001
- (5) Principal Investigator NSF grant no. DMS-0401084 for the period 2004-2007
- (6) Principal Investigator FRG grant no. DMS-0554247 for the period 2006-2011 awarded by NSF (A collaborative grant between UNC, Chapel Hill; University of Maryland; and University of California, Davis)
- (7) Principal Investigator NSF grant no. DMS-0901239 for the period 2009-2012
- (8) Invited 45 minute speaker at the International Congress of Mathematicians (ICM), Aug. 2010 in the section 'Lie Theory and Generalisations'
- (9) Principal Investigator NSF grant no. DMS-1201310 for the period 2012-2015

- (10) Fellow of the American Mathematical Society (chosen in 2012)
- (11) Grant from the United States - Israel Binational Science Foundation (jointly with M. Gorelik, A. Joseph and V. Serganova) for the period 2014-2017.
- (12) Principal Investigator NSF grant no. DMS-1501094 for the period 2015-2018.
- (13) Principal Investigator NSF grant no. DMS-1802328 for the period 2018-2021.

**Books Authored:**

- (1) *Kac-Moody Groups, Their Flag Varieties and Representation Theory*, Progress in Mathematics vol. 204, Birkhäuser, Boston, 606 pages (Aug. 2002).
- (2) (jt. with M. Brion) *Frobenius Splitting Methods in Geometry and Representation Theory*, Progress in Mathematics vol. 231, Birkhäuser, Boston, 250 pages (Dec. 2004).
- (3) *Conformal Blocks, Generalized Theta Functions and Verlinde Formula*, To be submitted soon.

**List of Research Publications (all refereed):**

- (1) A remark on universal connections. *Math. Ann.* 260, 453-462 (1982).
- (2) A G-minimal model for principal G-bundles. *Annales de L'Institut Fourier, Grenoble* 32, 205-219 (1982).
- (3) Geometry of Schubert cells and cohomology of Kac-Moody Lie algebras. *J. Diff. Geometry* 20, 389-431 (1984).
- (4) Rational homotopy theory of flag varieties associated to Kac-Moody groups. In: "Infinite dimensional groups with applications" (ed. by V. Kac). MSRI (Springer-Verlag) publications Vol. 4, 233-273 (1985).
- (5) Cohomology algebra of Schubert varieties associated to Kac-Moody groups. In: "Proceedings of the 1984 Vancouver Conference in Algebraic Geometry" (ed. by J. Carrell et. al.). CMS Conference Proceedings Vol. 6, 277-299 (1986).
- (6) A generalization of the Conner conjecture and topology of Stein spaces dominated by  $\mathcal{O}^n$ . *Topology* 25, 483-493 (1986).
- (7) (Jt. with B. Kostant) The nil Hecke ring and cohomology of G/P for a Kac-Moody group G. *Proc. Natl. Acad. Sci. USA* 83, 1543-1545 (1986).
- (8) A homology vanishing theorem for Kac-Moody algebras with coefficients in the category  $\mathcal{O}$ . *J. Algebra* 102, 444-462 (1986).
- (9) Non-representability of cohomology classes by bi-invariant forms (Gauge and Kac-Moody groups). *Comm. Math. Physics* 106, 177-181 (1986).
- (10) (Jt. with B. Kostant) The nil Hecke ring and cohomology of G/P for a Kac-Moody group G. *Advances in Math.* 62, 187-237 (1986).
- (11) (Jt. with B. Kostant) T-equivariant K-theory of generalized flag varieties. *Proc. Natl. Acad. Sci. USA* 84, 4351-4354 (1987).
- (12) Extension of the category  $\mathcal{O}^{\mathfrak{g}}$  and a vanishing theorem for the Ext functor for Kac-Moody algebras. *J. Algebra* 108, 472-491 (1987).
- (13) Demazure character formula in arbitrary Kac-Moody setting. *Invent. Math.* 89, 395-423 (1987).
- (14) Proof of the Parthasarathy-Ranga Rao-Varadarajan conjecture. *Invent. Math.* 93, 117-130 (1988).

- (15) Existence of certain components in the tensor product of two integrable highest weight modules for Kac-Moody algebras. In: "Infinite dimensional Lie algebras and groups" (ed. by V.G. Kac). Advanced series in Mathematical Physics Vol. 7, World Scientific, 25-38 (1989).
- (16) A refinement of the PRV conjecture. *Invent. Math.* 97, 305-311 (1989).
- (17) Bernstein-Gelfand-Gelfand resolution for arbitrary Kac-Moody algebras. *Math. Annalen* 286, 709-729 (1990).
- (18) (Jt. with K. Guruprasad) A new geometric invariant associated to the space of flat connections. *Compositio Math.* 73, 199-222 (1990).
- (19) (Jt. with B. Kostant) T-equivariant K-theory of generalized flag varieties. *J. Diff. Geometry* 32, 549-603 (1990).
- (20) Proof of Wahl's conjecture on surjectivity of the Gaussian map for flag varieties. In: "Algebraic groups" (ed. by S. Ramanan). Proceedings of the Hyderabad conference. Manoj Prakashan, 275-278 (1991).
- (21) (Jt. with B. Kostant) A geometric realization of minimal  $\mathfrak{k}$ -type of Harish-Chandra modules for complex s.s. groups. In: "Kazhdan-Lusztig theory and related topics" (ed. by V. Deodhar). Contemporary Mathematics vol. 139 published by AMS, 249-264 (1992).
- (22) Proof of Wahl's conjecture on surjectivity of the Gaussian map for flag varieties. *American J. of Math.* 114, 1201-1220 (1992).
- (23) (Jt. with V. Ginzburg) Cohomology of quantum groups at roots of unity. *Duke Math. J.* 69, 179-198 (1993).
- (24) Finiteness of local fundamental groups for quotients of affine varieties under reductive groups. *Comment. Math. Helvetici* 68, 209-215 (1993).
- (25) (Jt. with M. Vergne) Equivariant cohomology with generalized coefficients. *Astérisque* 215, 109-204 (1993).
- (26) Toward proof of Lusztig's conjecture concerning negative level representations of affine Lie algebras. *J. Algebra* 164, 515-527 (1994).
- (27) Symmetric and exterior powers of homogeneous vector bundles. *Math. Annalen* 299, 293-298 (1994).
- (28) Representations of quantum groups at roots of unity. In: "Quantum topology" (ed. by D.N. Yetter). World Scientific, Singapore, 187-224 (1994).
- (29) (Jt. with M. S. Narasimhan and A. Ramanathan) Infinite Grassmannians and moduli spaces of  $G$ -bundles. *Math. Annalen* 300, 41-75 (1994).
- (30) The nil Hecke ring and singularity of Schubert varieties. In: "Lie theory and geometry (in honor of B. Kostant)" (ed. by J.-L. Brylinski et. al.). Progress in Math. vol. 123, Birkhäuser, 497-507 (1994).
- (31) The nil Hecke ring and singularity of Schubert varieties. *Inventiones Math.* 123, 471-506 (1996).
- (32) (jt. with G. Letzter) Shapovalov determinant for restricted and quantized restricted enveloping algebras. *Pacific J. of Math.* 179, 123-161 (1997).
- (33) Infinite Grassmannians and moduli spaces of  $G$ -bundles. In: "Vector Bundles on Curves- New Directions," (ed. by M.S. Narasimhan). Springer Lecture Notes in Math. vol. 1649, 1-49 (1997).
- (34) (jt. with M.S. Narasimhan) Picard group of the moduli spaces of  $G$ -bundles. *Math. Annalen* 308, 155-173 (1997).

- (35) Fusion product of positive level representations and Lie algebra homology. In: “Geometry and physics” (ed. by J.E. Andersen et. al.). Lecture Notes in Pure and Applied Mathematics vol. 184, Marcel Dekker, Inc., 253-259 (1997).
- (36) (jt. with M.V. Nori) Positivity of the cup product in cohomology of flag varieties associated to Kac-Moody groups. *Inter. Math. Res. Not.* 14, 757-763 (1998).
- (37) (jt. with N. Lauritzen and J. Thomsen) Frobenius splitting of cotangent bundles of flag varieties. *Inventiones Math.* 136, 603–621 (1999).
- (38) Homology of certain truncated Lie algebras. In: “Recent Developments in Quantum Affine Algebras and Related Topics,” (ed. by K.C. Misra et. al.). *Contemporary Mathematics* vol. 248, 309–325 (1999), published by AMS.
- (39) (jt. with P. Littelmann) Frobenius splitting in characteristic zero and the quantum Frobenius map. *J. of Pure and applied Algebra* 152, 201–216 (2000).
- (40) (jt. with J.F. Thomsen) Frobenius splitting of Hilbert schemes of points on surfaces. *Math. Annalen* 319, 797–808 (2001).
- (41) (jt. with P. Littelmann) Algebraization of Frobenius splitting via quantum groups. *Annals of Mathematics* 155, 491–551 (2002).
- (42) (jt. with J.F. Thomsen) A conjectural generalization of the  $n!$  result to arbitrary groups. *Transformation Groups* 8, 69–94 (2003).
- (43) Equivariant analogue of Grothendieck’s theorem for vector bundles on  $\mathbb{P}^1$ . In: “Perspectives in Geometry and Representation Theory (A tribute to C.S. Seshadri),” Hindustan Book Agency, 500–501 (2003).
- (44) (jt. with B. Leeb and J. Millson) The generalized triangle inequalities for rank 3 symmetric spaces of noncompact type. *Contemporary Mathematics* vol. 332, published by AMS, 171–195 (2003).
- (45) (jt. with J.F. Thomsen) A new realization of the cohomology of Springer fibers. In: “Proceedings of the International Conference on Algebraic Groups and Arithmetic” (Volume dedicated to M.S. Raghunathan on his sixtieth birthday), Narosa Publishing House, 119–125 (2004).
- (46) Induction functor in noncommutative equivariant cohomology and Dirac cohomology. *J. Algebra* 291, 187–207 (2005).
- (47) (jt. with A. Boyzal) Explicit determination of the Picard group of moduli spaces of semistable  $G$ -bundles on curves. *Math. Annalen* 332, 823–842 (2005).
- (48) (jt. with K-H. Neeb) Extensions of algebraic groups. In: “Studies in Lie Theory” (Volume dedicated to A. Joseph on his sixtieth birthday), *Progress in Mathematics* vol. 243, Birkhäuser, 365–376 (2006).
- (49) (jt. with P. Belkale) Eigenvalue problem and a new product in cohomology of flag varieties. *Inventiones Math.* 166, 185–228 (2006).
- (50) (jt. with J. Stembridge) Special isogenies and tensor product multiplicities. *Inter. Math. Res. Not.* vol. 2007 (no. 20), 1–13 (2007).
- (51) On the Cachazo-Douglas-Seiberg-Witten conjecture for simple Lie algebras. *J. Am. Math. Soc.* 21, 797–808 (2008).
- (52) Descent of line bundles to GIT quotients of flag varieties by maximal torus. *Transformation Groups* 13, 757–771 (2008).
- (53) (jt. with W. Graham) On positivity in  $T$ -equivariant  $K$ -theory of flag varieties. *Inter. Math. Res. Not.* vol. 2008, 1–43 (August, 2008).
- (54) (jt. with M. Kapovich and J. Millson) The eigencone and saturation for  $\text{Spin}(8)$ . *Pure and Applied Math. Quarterly* 5, 755–780 (2009).

- (55) (jt. with G. Lusztig and D. Prasad) Characters of simplylaced nonconnected groups versus characters of nonsimplylaced connected groups. *Contemporary Math.* vol. 478, 99–101 (2009).
- (56) (jt. with A. Boysal) A conjectural presentation of fusion algebras. *Advanced Studies in Pure Math.* vol. 54 (Algebraic Analysis and Around), 95–107 (2009).
- (57) (jt. with V. Mehta) Finiteness of the number of compatibly split subvarieties. *Inter. Math. Res. Not.* vol. 2009 (no. 19), 3595–3597 (2009).
- (58) (jt. with P. Belkale) Eigencone, saturation and Horn problems for symplectic and odd orthogonal groups. *J. Alg. Geom.* 19, 199–242 (2010).
- (59) A generalization of Cachazo–Douglas–Seiberg–Witten conjecture for symmetric spaces. *Math. Annalen* 346, 67–84 (2010).
- (60) (non-refereed article) Tensor product decomposition. *Proceedings of the International Congress of Mathematicians, Hyderabad, India*, pages 1226–1261 (2010).
- (61) (jt. with P. Belkale and N. Ressayre) A generalization of Fulton’s conjecture for arbitrary groups. *Math. Annalen* 354, 401–425 (2012).
- (62) (jt. with C. Procesi) An algebro-geometric realization of equivariant cohomology of some Springer fibers. *J. Algebra* 368, 70–74 (2012).
- (63) An approach towards the Kollár–Peskine problem via the instanton moduli space. “Recent Developments in Lie Algebras, Groups, and Representation Theory”, PSPUM volume 86, 217–225 (2012).
- (64) (jt. with K. Schwede) Richardson varieties have Kawamata log terminal singularities. *Inter. Math. Res. Not.* vol. 2012 (issue 8), rns 241, 23 pages (2012).
- (65) Geometry of orbits of permanents and determinants. *Commentarii Math. Helv.* 88, 759–788 (2013).
- (66) (jt. with D. Prasad) Dimension of zero weight space: an algebro-geometric approach. *J. Algebra* 403, 324–344 (2014).
- (67) (jt. with N. Bushek) Hitchin’s conjecture for simply-laced Lie algebras implies that for any simple Lie algebra. *Diff. Geometry and its Applications* 35, 210–223 (2014).
- (68) (jt. with M. Brown) A study of saturated tensor cone for symmetrizable Kac-Moody algebras, *Math. Annalen* 360, 901–936 (2014).
- (69) A survey of the additive eigenvalue problem (with an appendix by M. Kapovich), *Transformation Groups* 19, 1051–1148 (2014).
- (70) A study of the representations supported by the orbit closure of the determinant, *Compositio Math.* 151, 292–312 (2015).
- (71) (jt. with J.M. Landsberg) Connections between conjectures of Alon-Tarsi, Hadamard-Howe, and integrals over the special unitary group, *Discrete Mathematics* 338, 1232–1238 (2015).
- (72) Additive eigenvalue problem, *EMS Newsletter*, December 2015, 20–27 (2015).
- (73) (jt. with P. Belkale) The multiplicative eigenvalue problem and deformed quantum cohomology. *Advances in Math.* 288, 1309–1359 (2016).
- (74) Representation ring of Levi subgroups versus cohomology ring of flag varieties, *Math. Annalen* 366, 395–415 (2016).
- (75) (jt. with R. Chirivì and A. Maffei) Components of  $V(\rho) \otimes V(\rho)$ , *Transformation Groups* 22, 645–650 (2017).
- (76) Positivity in  $T$ -equivariant  $K$ -theory of flag varieties associated to Kac-Moody groups (with an appendix by M. Kashiwara), *J. Eur. Math. Soc.* 19, 2469–2519 (2017).

- (77) (jt. with S. Baldwin) Positivity in  $T$ -equivariant  $K$ -theory of flag varieties associated to Kac-Moody groups II, *Representation Theory* (An electronic journal of the American Mathematical Society) 21, 35–60 (2017).
- (78) A complete set of intertwiners for arbitrary tensor product representations via current algebras, *Transformation Groups* 24, 115–125 (2019).
- (79) (jt. with N. Ressayre) On the faces of the tensor cone of symmetrizable Kac-Moody Lie algebras, Preprint on the ArXiv, 61 pages (2019).
- (80) (jt. with J. Hong) Conformal blocks for Galois covers of algebraic curves, Preprint on the ArXiv, 72 pages (2019).

#### Some Notable Invited Talks/Series of Lectures:

- (1) A series of eight lectures at the **C.I.M.E. session on “Vector bundles on curves. New directions”** held in Cetraro (Italy), June 1995.
- (2) An hour address at the **American Math Society spring sectional meeting** held at Davidson College, March 2007.
- (3) **CRM-University of Ottawa Distinguished Lecture** at the University of Ottawa, April 2007.
- (4) Gave a course consisting of 10 hours of lectures on ‘Demazure character formula in arbitrary Kac-Moody setting and its consequences’ at the **Taipei Winter School in Lie Theory** held in Taipei, December 2008.
- (5) Gave a course consisting of 10 hours of lectures on **Eigenvalue problem for reductive groups** at RIMS, Kyoto (Japan), May 2009.
- (6) Gave a course consisting of 4.5 hours of lectures on **Infinite Grassmannians, moduli spaces of  $G$ -bundles and Verlinde formula** at the University of Georgia, Athens (USA), VIGRE 2010 Summer School Workshop, May 2010.
- (7) Gave a course consisting of 2 hours of lectures on **Introduction to Frobenius splitting** at the ‘Frobenius Splitting in Algebraic Geometry, Commutative Algebra, and Representation Theory’ conference held at the University of Michigan, Ann Arbor (USA), May 2010.
- (8) Gave a course consisting of 10 hours of lectures on **Geometry of Schubert varieties and Demazure character formula** at the Hausdorff Research Institute for Mathematics, Bonn (Germany) during April 2011.
- (9) Gave an hour plenary address at the **DMV Annual Conference, 2011**, Cologne (Germany), September 2011.
- (10) Gave an hour **Spring 2012 Distinguished Lecture** at the University of Massachusetts, Amherst, February 2012.
- (11) Gave four hours’ mini-course on **equivariant  $K$ -theory of flag varieties** at Jacobs University, Bremen (Germany), August 2012.
- (12) Gave four one and a half hours’ mini-course on **Eigenvalue /Saturated Tensor Product Problem for Reductive Groups** at the University of Pisa, Pisa (Italy), June 2014.
- (13) Gave three two hours’ mini-course on **Conformal blocks and Verlinde formula** at l’Université Claude Bernard Lyon 1, Lyon (France), June 2017.
- (14) Gave three one and a half hours’ mini-course on **Space of Conformal Blocks and Proof of Verlinde Dimension Formula** at Tata Institute of Fundamental Research (Mumbai), January 2018.

- (15) Gave three one hour mini-course on **Littlewood-Richardson cone for symmetrizable Kac-Moody algebras** at Physikzentrum Bad Honnef (Germany) during the conference ‘Geometry and Representation Theory of Algebraic Groups’ celebrating the sixtieth birthday of F. Knop and P. Littelmann, March 2018.

**Invited Talks:** Gave the invited talks in the following conferences.

- (1) **Infinite dimensional groups with applications** held at MSRI, Berkeley (USA), May 1984.
- (2) **Algebraic geometry** held at The University of British Columbia, Vancouver (Canada), July 1984.
- (3) **Infinite dimensional Lie algebras and Lie groups** held at CIRM, Luminy (France), July 1988.
- (4) AMS meeting on **Infinite-dimensional symmetries in mathematics and physics** held in Worcester, MA (USA), April 1989.
- (5) AMS meeting on **Kazhdan-Lusztig theory and related topics** held in Chicago (USA), May 1989.
- (6) International conference on **Algebraic groups and applications** held at the University of Hyderabad (India), Dec. 1989.
- (7) **Enveloping algebras and rings of differential operators** held at Oberwolfach (Germany), April 1990.
- (8) Indo-USSR conference on **Geometry** held at T.I.F.R., Bombay (India), Jan. 1991.
- (9) The annual AMS meeting (**Quantum Groups** session) held in San Antonio, Texas (USA), Jan. 1993.
- (10) Symposium in honor of B. Kostant **Lie theory, algebra, and geometric quantization** held at M.I.T., Cambridge (USA), May 1993.
- (11) **Vector bundles on curves- New directions** held at T.I.F.R. (Bombay) and SPIC Science Foundation (Madras), Dec. 1993.
- (12) **Moduli spaces of semistable bundles over Riemann surfaces** held at University of Aarhus, Aarhus (Denmark), May 1995.
- (13) AMS meeting on **Quantum Kac-Moody Lie algebras and related topics** held in Greensboro, NC (USA), Nov. 1995.
- (14) **Schubert varieties; Geometry, algebra and combinatorics** held at Oberwolfach (Germany), April 1997.
- (15) **Algebraic and differential geometry** held at T.I.F.R. (Bombay), July 1997.
- (16) **Einhüllende algebren und darstellungstheorie** held at Oberwolfach (Germany), November 1997.
- (17) **Quantum groups and applications** held at Manhattan (Kansas), March 1998.
- (18) **Representations of affine and quantum affine algebras and related topics** held at NCSU, Raleigh, NC (USA), May 1998.
- (19) **Representation theory for algebraic groups and quantum groups** held at Aarhus (Denmark), August 1998.
- (20) **Enveloping algebras** held at The Weizmann Institute of Science, Rehovot (Israel), April 1999.
- (21) **Théorie des groupes** held at Institut de Mathématiques de Jussieu, Paris (France), June 1999.
- (22) **Workshop on nonabelian theta functions and moduli spaces of vector bundles** held in Salamanca (Spain), June 1999.



- (23) **Representations of algebraic groups** held in Luminy (France), November 1999.
- (24) **Mini-conference on Lie theory and geometry** held in Tucson (Arizona), December 1999.
- (25) **Algebraic groups and representation theory** held in Odense (Denmark), June 2000.
- (26) **Algebraic groups, invariant theory and applications** held in Wien (Austria), August 2000.
- (27) **Infinite dimensional Lie algebras and groups: Structure and geometry** held in Toronto (Canada), September 2000.
- (28) **Mini conference** held at Aarhus (Denmark), April 2001.
- (29) **Solstice d'ete on the theme 'Group Representations'** held in Paris (France), June 2001.
- (30) **International conference on representations of algebraic groups and quantum groups** held in Kunming (China), July 2001.
- (31) **International conference on algebraic groups and arithmetic** held in Mumbai (India), December 2001.
- (32) **Enveloping algebras and algebraic Lie representations** held at Oberwolfach (Germany), April-May 2002.
- (33) **International conference on Lie and Jordan algebras, their representations and applications** held at Guarujá, Sao Paulo (Brazil), May 2002.
- (34) **Algebraic transformation groups** held at Montreal (Canada), June 2002.
- (35) **A colloquium on geometry in honour of M.S. Narasimhan on the occasion of his 70th birthday** held at ICTP, Trieste (Italy), December 2002.
- (36) **Conference on representations of algebraic groups** held at Aarhus (Denmark), June 2003.
- (37) **Infinite dimensional Lie theory workshop** held in Toronto (Canada), July 2003.
- (38) **International colloquium on algebraic groups and homogeneous spaces** held in Mumbai (India), January 2004.
- (39) **Cohomological aspects of Hamiltonian group actions and toric varieties** held at Oberwolfach (Germany), April 2004.
- (40) **Infinite dimensional aspects of representation theory and applications** held at University of Virginia, Charlottesville (USA), May 2004.
- (41) **International conference on representation theory III** held at Sichuan University, Chengdu (China), August 2004.
- (42) **Interaction of finite dimensional algebras with other areas of mathematics** held at Banff International Research Station for Mathematical Innovation and Discovery (Canada), September 2004.
- (43) **Geometric representation theory** held at University of Arizona (USA), March 2005.
- (44) **Applications of torsors to Galois cohomology and Lie theory** held at Banff International Research Station for Mathematical Innovation and Discovery (Canada), April 2005.
- (45) **Algebraic combinatorics and geometry** held at University of Oregon (USA), November 2005.
- (46) **Lie algebras and representation theory**, American Math Society and Taiwanese Math Society joint meeting held at Tunghai University, Taichung (Taiwan), December 2005.

- (47) **Texas algebraic geometry seminar** held at Texas A and M University (USA), May 2006.
- (48) **Geometry and representation theory** (in honor of sixtieth birthday of G. Lusztig) held at MIT (USA), June 2006.
- (49) **Geometry and representations** held in Luminy (France), December 2006.
- (50) **Infinite dimensional Lie theory** held at Oberwolfach (Germany), December 2006.
- (51) **Algebraic groups** held at Oberwolfach (Germany), April 2007.
- (52) **Representations of algebraic groups and related topics** held in Bielefeld (Germany), April 2007.
- (53) **Algebraic analysis and around** (on the occasion of M. Kashiwara's sixtieth birthday) held at RIMS, Kyoto (Japan), June 2007.
- (54) **International conference on representation theory IV** held in Lhasa (Tibet), July 2007.
- (55) **B-stable ideals and nilpotent orbits** held at Istituto Nazionale di Alta Matematica, Rome (Italy), October 2007.
- (56) **Transformation Groups** (on the occasion of E. Vinberg's seventieth birthday) held at the Independent University of Moscow, Moscow (Russia), December 2007.
- (57) **John Millson 62: Lie theory, geometry, and discrete groups** held at University of Maryland, College Park, March 2008.
- (58) **AMS - SBM joint international meeting** held at IMPA, Rio de Janeiro (Brazil), June 2008.
- (59) **Conference on vector bundles in honour of S. Ramanan** (on the occasion of his seventieth birthday) held in Madrid (Spain), June 2008.
- (60) AMS meeting on **Kac-Moody algebras, vertex algebras, quantum groups, and applications** held at NCSU, Raleigh, April 2009.
- (61) **Eigenvalue and saturation problems for reductive groups** held at the University of North Carolina, Chapel Hill, May 2009.
- (62) **Representation theory** held at the University of Utah, Salt Lake City, July 2009.
- (63) **Lie algebras, vertex algebras and automorphic forms** held at ICMS, Edinburgh (Scotland), September 2009.
- (64) **CBMS conference on quiver varieties and quantum affine algebras** held at NCSU, Raleigh, May 2010.
- (65) **MAA Mathfest conference on combinatorial games and Schubert calculus** held in Pittsburgh, PA, August 2010.
- (66) **Infinite dimensional Lie theory** held at Oberwolfach (Germany), November 2010.
- (67) **UW-UBC algebraic geometry seminar** held at University of British Columbia (Vancouver), March 2011.
- (68) **The third SE Lie theory workshop** held at University of Virginia, Charlottesville (USA), June 2011.
- (69) **Mathematical aspects of P versus NP and its variants** held at ICERM (Providence), August 2011.
- (70) **First Latin American school on algebraic geometry** held in La Cumbre, Cordoba (Argentina), August 2011.
- (71) **DMV Annual Conference, Algebra Section** held in Cologne (Germany), September 2011.
- (72) **Geometry and representation theory related to geometric complexity and other variants of P v. NP** held at Snowbird, Utah, June 2012.

- (73) **Geometric and Algebraic Aspects of Representation Theory**, AMS Southeastern Fall Section Meeting held in New Orleans (USA), October 2012.
- (74) **International Conference on Algebraic Geometry in honor of Prof. M.S. Narasimhan's eightieth birthday** held in Bangalore (India), December 2012.
- (75) **AMS Special Session on Geometric Complexity Theory (a MRC Session)**, Joint Mathematics Meetings held in San Diego, January 2013.
- (76) **Algebraic groups** held at Oberwolfach (Germany), April 2013.
- (77) **Algebra, Geometry and Combinatorics Day** at IUPUI, Indianapolis, April 2013.
- (78) **The Third International Symposium on Groups, Algebras and related topics** celebrating the 50th anniversary of the Journal of Algebra held at the Peking University, Beijing, June 2013.
- (79) **The 6th International Conference on Representation Theory (ICRT-VI)** held in Zhangjiajie (Hunan Province, China), June 2013.
- (80) Plenary talk in the conference **Differential Geometry and its Applications** held in Brno (Czech Republic), August 2013.
- (81) **Representations of Lie algebras and Lie superalgebras** held at the University of Georgia, Athens, May 2014.
- (82) **Representation theory of algebraic groups** held at the Universite de Lyon, Lyon (France), July 2014.
- (83) **Representations of algebraic groups and related objects** held at Friedrich-Schiller-University, Jena (Germany), September 2014.
- (84) **Geometric Complexity Theory** held at the Simons Institute for the Theory of Computing, Berkeley, September 2014.
- (85) **Aspects of Lie Theory** held at the Istituto Nazionale di Alta Matematica, Rome (Italy), January 2015.
- (86) **Representation Theory Workshop** held at Uppsala University, Uppsala (Sweden), May 2015.
- (87) **Enveloping Algebras and Geometric Representation Theory** held at Oberwolfach (Germany), May 2015.
- (88) **Representation Theory and Algebraic Geometry (special session of the 2015 AustMS Conference)** held at Flinders University (Australia), September 2015.
- (89) **Taipei Conference in Representation Theory V (dedicated to George Lusztig on the occasion of his 70th birthday)** held at Institute of Math, Academia Sinica, Taipei (Taiwan), January 2016.
- (90) **Texas A&M Regional Conference** held at Texas A&M (Texas), November 2016.
- (91) **Algebraic Groups, TIMC-AMS Conference** held in Benares, India, December 2016.
- (92) **Algebraic Groups** held at Oberwolfach (Germany), April 2017.
- (93) **Algèbre, Rencontres Grenoble/Lyon/Saint-Étienne** held in Lyon, France, June 2017.
- (94) **Algebraic Modes of Representations - The Canicular Days** held at the Weizmann Institute, Rehovot, Israel, July 2017.
- (95) **Representation Theory, Geometry, and Quantization: the mathematical legacy of Bertram Kostant** held at MIT, May 2018.
- (96) **Interactions of Quantum Affine Algebras with Cluster Algebras, Current Algebras and Categorification (in honor of V. Chari's 60th Birthday)** held at the Catholic University of America, Washington D.C., June 2018.

- (97) **Geometric Quantization and Applications** held at CIRM, Luminy, France, October 2018.
- (98) **Algebraic Groups: Geometry, Actions and Structure (Dedicated to M. Brion on his 60th birthday)** held at Université Claude Bernard, Lyon, France, October-November 2018.
- (99) **Enveloping Algebras and Geometric Representation Theory** held at Oberwolfach (Germany), November 2018.
- (100) **Conference in Geometry and Representations (in celebration of W. Haboush's 75th birthday)** held in Jeju Island, South Korea, January 2019.

### Professional Services

- (1) Coorganizer for the International conference on **Algebraic groups and applications** held at the University of Hyderabad (India), Dec. 1989.
- (2) Served as an editor in the journal "Communications in Algebra", for the period Jan. 1990- Dec. 94.
- (3) Coorganizer for the Indo-USSR conference on **Geometry** held at T.I.F.R., Bombay (India), Jan. 1991.
- (4) Coorganizer for the Symposium in honor of B. Kostant **Lie theory, algebra, and geometric quantization** held at M.I.T., Cambridge (USA), May 1993.
- (5) Reviewer for Zentralblatt für Mathematik.
- (6) Member of the editorial board for the Lecture note series PRAVESHKA published by the National Board of Higher Mathematics, India.
- (7) Coorganizer for the conference on **Algebraic and differential geometry** held in July 1997 in Bombay.
- (8) Member of the editorial board of the journal "Transformation Groups" since 1998.
- (9) Director of Graduate Studies, UNC at Chapel Hill, July 1997- June 2000.
- (10) Member of the editorial board of the 'Advanced texts in mathematics' series of Birkhauser, 2001- -.
- (11) Member of the editorial board for the publication of 'Collected papers of B. Kostant' to be published by Springer-Verlag.
- (12) Member of the UNC Math Department Advisory Committee (an elected body), 2003- 2006.
- (13) Coorganizer for the meeting on **Enveloping algebras and geometric representation theory** held at Oberwolfach (Germany), March 2005.
- (14) Coorganizer for the meeting on **Applications of torsors to Galois cohomology and Lie theory** held at Banff International Research Station for Mathematical Innovation and Discovery (Canada), April 2005.
- (15) Served on the review panel of **Science Foundation Ireland**, November 2006 and again March 2007.
- (16) Coorganizer for the meeting on **Lie theory and geometry: the mathematical legacy of Bertram Kostant** held at the Pacific Institute of Mathematical Sciences, Vancouver (Canada), May 2008.
- (17) Coorganizer for the meeting on **Enveloping algebras and geometric representation theory** held at Oberwolfach (Germany), March 2009.
- (18) Coorganizer for the meeting on **Eigenvalue and saturation problems for reductive groups** held at UNC, Chapel Hill, May 2009.

- (19) Served on the NSF Career Core panel, October 2009; NSF review panel, February 2017.
- (20) Coorganizer for the meeting on **Frobenius splitting in algebraic geometry, commutative algebra, and representation theory** held at the University of Michigan, Ann Arbor, May 2010.
- (21) Member of the editorial board of the journal "Journal of Algebra" July 2010 – January 2018.
- (22) Coorganizer for the meeting on **Enveloping algebras and geometric representation theory** held at Oberwolfach (Germany), March 2012.
- (23) Coorganizer for the MRC summer conference on **Geometry and representation theory related to geometric complexity and other variants of P v. NP** held at Snowbird, Utah, June 2012.
- (24) Director of Graduate Studies, UNC at Chapel Hill, July 2011- June 2012.
- (25) Member of the editorial board of 'Journal of the Ramanujan Math Society', 2014 --.
- (26) Coorganizer for the **Workshop on Moduli Spaces, Derived Geometry, and Geometric Representation Theory** held at UNC, Chapel Hill, November 2014.
- (27) Coorganizer for the **AMS Sectional Meeting (special Session on Lie Theory, Representation Theory, and Geometry)** held at University of Georgia, Athens (GA), March 2016.
- (28) Coorganizer for the **AMS Sectional Meeting (special Session on Algebraic Geometry, Representation Theory, and Applications)** held at Vanderbilt University, Nashville (TN), April 2018.
- (29) Coorganizer for the meeting on **Representation Theory, Geometry, and Quantization: The Mathematical Legacy of Bertram Kostant** held at the Massachusetts Institute of Technology, Cambridge (MA), May 2018.
- (30) Coorganizer for the **AMS Sectional Meeting (special Session on Geometric Methods in Representation Theory)** held at Auburn University, Auburn (AL), March 2019.
- (31) Associate Chair, Department of Mathematics, UNC, July 2016 - -

#### **Students Completed Ph. D. Under My Supervision**

- (1) David W. Lyons (Fall, 1996)
- (2) Richard H. Hammack (Fall, 1996)
- (3) S. Ilangovan (Fall, 1998)
- (4) Shawn Robinson (Spring, 2001)
- (5) Arzu Boysal (Spring, 2005)
- (6) Charles Hague (Summer, 2007)
- (7) Brandyn Lee (Spring, 2012)
- (8) Merrick Brown (Summer, 2014)
- (9) Nathaniel Bushek (Spring, 2015)
- (10) Seth Baldwin (Spring, 2018)
- (11) Sean Rogers (Fall, 2018)

#### **Students Completed M. S. Under My Supervision**

- (1) Swarnava Mukhopadhyay (Spring, 2010)

### A Brief Summary of Body of My Works:

My main interests lie in Representation Theory of finite dimensional semisimple groups and their Kac-Moody analogs and the geometry and topology of their flag varieties. In addition, I have been interested in the moduli of semistable principal  $G$ -bundles over curves in its connection to Verlinde formula for the dimension of the space of conformal blocks and also the  $G$ -analog of the classical Hermitian eigenvalue problem, where  $G$  is any complex semisimple group.

The following is a brief description of some of my main results.

I extended the Laplacian calculation of Kostant (in the finite case) and Garland (in the affine case) to any symmetrizable Kac-Moody algebras and used this to develop a Hodge theory for their flag varieties (1984).

Motivated by Hodge theory, I jointly with Kostant introduced a ring (now commonly known as the *Kostant-Kumar nil-Hecke ring* or just the *nil-Hecke ring*) and used this to give a purely algebraic model for the cohomology of flag varieties associated to any semisimple group, more generally, any Kac-Moody group (1986). In particular, we gave an expression for the cup product of any two Schubert cohomology classes. This is the only known ‘explicit’ formula for the cup product which works in general. This work has been used in several important works including in uniformly determining the quantum cohomology of flag varieties. (Arabia extended this work to the equivariant cohomology in 1989.)

Jointly with Kostant I obtained similar results for the equivariant  $K$ -theory of any flag variety (1990). Again, this work has extensively been used in later developments in the subject. More recently, I came back to the study of equivariant  $K$ -theory of flag varieties and made a ‘positivity’ conjecture jointly with Graham (2008) for the dual Schubert basis. This conjecture has been established by Anderson-Griffeth-Miller (2011) in the finite case. In the general symmetrizable Kac-Moody case, the conjecture has been established by me (2017). I also proved an analogous positivity result for the Schubert basis jointly with Baldwin in the general symmetrizable Kac-Moody case (2017). In particular, this settles a conjecture due to Lam-Schilling-Shimozono (2010).

I proved the Demazure character formula for an arbitrary Kac-Moody group  $G$  by a new algebro-geometric method (1987). Recall that the Demazure character formula explicitly gives the character of the  $B$ -submodule generated by any extremal weight vector in an integrable highest weight  $G$ -module, where  $B$  is a Borel subgroup of  $G$ . This work has numerous applications, e.g., this is fundamental to the proof of Verlinde formula (see below). Moreover, I used this character formula to extend the celebrated Weyl-Kac character formula for symmetrizable Kac-Moody algebras to an arbitrary (not necessarily symmetrizable) Kac-Moody algebras.

In the nineteen sixties, Parthasarathy-Ranga Rao-Varadarajan gave an important conjecture on the decomposition of tensor product of two representations. Their conjecture asserted that for any two irreducible  $G$ -modules  $V(\lambda), V(\mu)$  with highest weights  $\lambda, \mu$  respectively, and any Weyl group element  $w$ , the tensor product  $V(\lambda) \otimes V(\mu)$  has a component with extremal weight  $\lambda + w\mu$ , where  $G$  is a complex semisimple group. Subsequently, Kostant strengthened this conjecture by identifying the ‘first occurrence’ of this piece in the tensor product. Now, I proved this conjecture by employing a mixture of algebro-geometric and representation theoretic techniques (1988). This result has found many applications by several mathematicians in different areas.

I (in collaboration with Ginzburg) determined the cohomology of quantized enveloping algebras at roots of unity (1993). This result has been used in several subsequent works including a fundamental work by Arkhipov-Bezrukavnikov-Ginzburg.

I (jointly with Vergne) introduced and studied the equivariant cohomology of manifolds with generalized coefficients (1993). We proved a version of Kunnet theorem and localization theorem for this new cohomology. This new cohomology has been used by others, especially in the study of the index of transversally elliptic operators.

A very important conjectural formula came out of Mathematical Physics, known as the Verlinde formula given by E. Verlinde (1988). The Verlinde formula attracted a lot of attention from mathematicians when it was heuristically realized that for the Wess-Zumino-Witten model associated to a simple algebraic group  $G$  over  $\mathbb{C}$  (which is a particular Rational Conformal Field Theory), the space of conformal blocks admits an interpretation as the space of generalized theta functions, which is the space of holomorphic sections of the theta bundle on the moduli space  $\mathcal{M}_G(\Sigma)$  of semistable principal  $G$ -bundles on a smooth projective curve  $\Sigma$ . This interpretation was rigorously established by Kumar-Narasimhan-Ramanathan (1994) (and also independently by Beauville-Laszlo and Faltings). Now, by a result of Tsuchiya-Ueno-Yamada, the dimension of the space of conformal blocks is given by the Verlinde formula. Thus, by the above result of Kumar-Narasimhan-Ramanathan (and others), Verlinde's conjectural formula for the dimension of the space of generalized theta functions gets established. A Séminaire Bourbaki talk by C. Sorger on 'La formule de Verlinde' during November, 1994 was devoted to these works. Building upon the above work, I (jointly with Narasimhan in 1997 and in another paper with Boysal in 2005) determined precisely the Picard group of the projective variety  $\mathcal{M}_G(\Sigma)$ . Recently (2018), I jointly with Hong have begun a systematic study to extend the theory of conformal blocks to a 'twisted' setting where the curve  $\Sigma$  is replaced by a finite Galois cover of  $\Sigma$  and the affine Kac-Moody group by the twisted affine Kac-Moody group. Specifically, we study the space of twisted conformal blocks attached to  $A$ -curves with marked  $A$ -orbits and an action of  $A$  on a simple Lie algebra  $\mathfrak{g}$ , where  $A$  is a finite group. We prove that if  $A$  stabilizes a Borel subalgebra of  $\mathfrak{g}$ , then Propagation Theorem and Factorization Theorem hold. We endow a projectively flat connection on the sheaf of twisted conformal blocks attached to a smooth family of pointed  $A$ -curves; in particular, it is locally free. We also prove that the sheaf of twisted conformal blocks on the stable compactification of Hurwitz stack is locally free. In a continuation of this work (still in progress) we connect the space of twisted conformal blocks with the global sections of certain line bundles on a moduli stack.

I determined the precise singular locus of any Schubert variety in any flag variety in terms of the nil-Hecke ring (1996). Partial results in this direction were given earlier by several mathematicians. However, my criterion is a uniform criterion to determine such a locus and it has widely been used. It was extended by Juteau-Williamson in char.  $p$ .

An important breakthrough in understanding the geometry of Schubert varieties was the introduction by Mehta-Ramanathan of the notion of Frobenius split varieties and the result that the flag varieties  $G/P$  are Frobenius split. I (jointly with Lauritzen and Thomsen) have shown that the cotangent bundle of the flag varieties is Frobenius split for any good prime  $p$  (1999). This has provided several uniform and sharp results in the area.

In addition, I (in collaboration with Littelmann) have given a complete and self-contained representation theoretic approach to the Frobenius splitting method for  $G/P$  (2002). This new approach provides the Frobenius splitting very explicitly at the level of representations. The geometric Frobenius method (in char.  $k = p > 0$ ) has been replaced by Lusztig's

Frobenius maps for quantum groups at roots of unity (which exist not only for primes but any integer  $\ell > 1$ ).

I (jointly with Thomsen) gave a conjectural generalization (2003) of the famous  $n!$  theorem due to Haiman.

The classical Hermitian eigenvalue problem asks the possible eigenvalues of the sum  $A + B$  of two Hermitian matrices  $A$  and  $B$  under the constraint that the eigenvalues of  $A$  and  $B$  are fixed. The first nontrivial result towards this problem was obtained by H. Weyl (1912). This problem continued to attract the attention of several mathematicians during the last century and it was finally solved by combining the works of Horn (1962), Klyachko (1998), Knutson-Tao (1999) and Belkale (2001). This work was generalized for other complex reductive groups by Berenstein-Sjamaar (2000) and Kapovich-Leeb-Millson (2005). However, their work did not provide an optimal solution to the problem. Their system of inequalities had redundancies for any group  $G$  of type different from  $A_n$ . Now, I (jointly with Belkale) came up with a new product in the cohomology of flag varieties (now known as the *Belkale-Kumar product*) and used this to give a (in general much smaller) set of inequalities solving the eigenvalue problem (2006). It was shown by Ressayre that these smaller set of inequalities given by Belkale-Kumar provides an optimal solution of the eigenvalue problem for any reductive group  $G$  (2010). Thus, in some sense, my work with Belkale concluded this fundamental problem at a theoretical level, though an ‘explicit’ determination of the eigencone for general  $G$  is not yet fully achieved. Vergne, Berline and Walter have made some progress in this direction. Continuing our work, I (in collaboration with P. Belkale) determined the eigencone for the symplectic and odd orthogonal groups in terms of that of the ambient special linear group (2010). Moreover, I (jointly with Belkale and Ressayre) extended Fulton’s conjecture to an arbitrary group (2011). There was a Séminaire Bourbaki talk in November, 2011 by M. Brion on ‘Restriction de représentations et projection d’orbites co-adjointes’ (d’après Belkale, Kumar, Ressayre) covering some of these works. I (jointly with Belkale) also solved the corresponding multiplicative eigenvalue problem for any compact group by giving an optimal set of inequalities to determine the corresponding polytope in terms of the quantum cohomology of flag varieties (and its deformed version introduced by us) (2016).

Cachazo-Douglas-Seiberg-Witten gave a conjecture on the structure of the conformal algebra (associated to any complex simple Lie algebra  $\mathfrak{g}$ ) arising in Supersymmetric Gauge Theory (2002). They proved their conjecture for the special linear Lie algebra  $\mathfrak{g} = \mathfrak{sl}_N$  and subsequently Witten and Etingof proved the conjecture for other classical Lie algebras. Now, I proved a substantial part of the conjecture for any simple  $\mathfrak{g}$  (2008). I came up with a uniform proof for any simple Lie algebra  $\mathfrak{g}$  using the geometry and topology of loop groups. I further generalized the result to the symmetric spaces (2010).

In an attempt to understand Valiant’s conjecture and its strengthened version due to Mulmuley-Sohoni in Geometric Complexity Theory, I studied the geometry of the orbit closure of determinant and permanent and showed that they are *not* normal (2013). I further studied the representations supported by these orbit closures and connected its study to a famous ‘Latin Square Conjecture’ from 1992 due to Alon-Tarsi (2015).

I began a study to connect the cohomology of flag varieties  $G/P$  under cup product with the representation ring of  $L$ , where  $G$  is any semisimple group and  $P$  is any parabolic subgroup with  $L$  as its Levi component (2016). This provides a substantial generalization of the classical result connecting the cohomology algebra of the Grassmannians with the representation ring of general linear groups.



In a joint work with Brown, I initiated a study of the saturated tensor cone for the integrable highest weight modules of symmetrizable Kac-Moody Lie algebras (2014). We solved the problem for affine  $SL(2)$  and made some conjectures in general. Some of these conjectures have been established by Ressayre (2017). In a work in progress (2018), I jointly with Ressayre have proved an irredundance result for the saturated tensor cone for any symmetrizable Kac-Moody Lie algebra.

I have authored two books: *Kac-Moody groups, their flag varieties and representation theory* (2002) and the other jointly with Brion on *Frobenius splitting methods in geometry and representation theory* (2004). Both of these are the very first books on the subject and they have become standard references.